

ARITHMETIC SEQUENCES. Find the next few terms in the sequence and then find the requested term.

1) 2, 4, 6, 8, 10, 12, 14,

Find $a_{42} = 42(2) = 84$

$$\begin{aligned} 2+2 &= 4 \\ 2+2+2 &= 6 \\ 2+2+2+2 &= 8 \\ 2+2+2+2+2 &= 10 \\ 2+2+2+2+2+2 &= 12 \\ 2+2+2+2+2+2+2 &= 14 \end{aligned}$$

$$\begin{aligned} a_1 &= 1(2) = 2 \\ a_2 &= 2(2) = 4 \\ a_3 &= 3(2) = 6 \\ a_4 &= 4(2) = 8 \\ a_5 &= 5(2) = 10 \\ a_6 &= 6(2) = 12 \\ a_7 &= 7(2) = 14 \end{aligned}$$

Current TERM Previous TERM

Determine the RECURSIVE DEFINITION:

$$a_n = a_{n-1} + 2$$

Determine the EXPLICIT DEFINITION:

$$a_n = 2n$$

2) 5, 8, 11, 14, 17, 20, 23,

Find $a_{33} = 101$

$$\begin{aligned} 5 &= 5 \\ 5+3 &= 8 \\ 5+3+3 &= 11 \\ 5+3+3+3 &= 14 \\ 5+3+3+3+3 &= 17 \\ 5+3+3+3+3+3 &= 20 \\ 5+3+3+3+3+3+3 &= 23 \end{aligned}$$

$$\begin{aligned} a_1 &= 5+0(3) = 5 \\ a_2 &= 5+1(3) = 8 \\ a_3 &= 5+2(3) = 11 \\ a_4 &= 5+3(3) = 14 \\ a_5 &= 5+4(3) = 17 \\ a_6 &= 5+5(3) = 20 \\ a_7 &= 5+6(3) = 23 \end{aligned}$$

Determine the RECURSIVE DEFINITION:

$$a_n = a_{n-1} + 3$$

Determine the EXPLICIT DEFINITION:

$$a_n = 5 + (n-1) \cdot 3$$

3) 10, 7, 4, 1, -2, -5, -8,

Find $a_{29} = -74$

$$\begin{aligned} 10 &= 10 \\ 10-3 &= 7 \\ 10-3+3 &= 4 \\ 10-3+3+3 &= 1 \\ 10-3+3+3+3 &= -2 \\ 10-3+3+3+3+3 &= -5 \\ 10-3+3+3+3+3+3 &= -8 \end{aligned}$$

$$\begin{aligned} a_1 &= 10+0(-3) = 10 \\ a_2 &= 10+1(-3) = 7 \\ a_3 &= 10+2(-3) = 4 \\ a_4 &= 10+3(-3) = 1 \\ a_5 &= 10+4(-3) = -2 \\ a_6 &= 10+5(-3) = -5 \\ a_7 &= 10+6(-3) = -8 \end{aligned}$$

Determine the RECURSIVE DEFINITION:

$$a_n = a_{n-1} - 3$$

Determine the EXPLICIT DEFINITION:

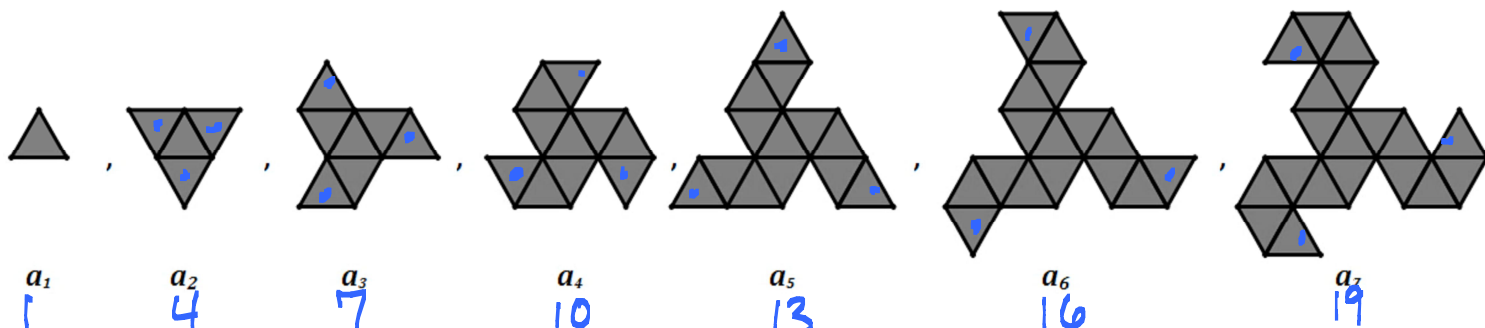
$$a_n = 10 + (n-1) \cdot (-3)$$

FORMULA: $a_n = a_1 + (n-1)d$ ← COMMON DIFFERENCE

• CURRENT TERM FIRST TERM NUMBER OF TERM

4) Josh was making a sequence pattern out of triangle pattern blocks.

ARITHMETIC SEQUENCE



If Josh continues this pattern, how many triangles will he need to make the 20th step of this pattern?

$$a_n = a_1 + (n-1)d$$

$$a_{20} = (1) + (20-1)3$$

$$a_{20} = 1 + (19)3 = 1 + 57 = \boxed{58}$$

Functions can be used as explicit definitions for a sequence:

Consider the sequence: 4, 7, 10, 13, 16, 19, 22, 25, The function $f(x) = 4 + (x-1)3$ could be used to define the sequence where x = the term number. The domain would be $\{1, 2, 3, 4, \dots\}$ and the range would be $\{4, 7, 10, 13, \dots\}$

5) Create a sequence based on the function: $f(x) = 4x - 1$ DOMAIN: $\{1, 2, 3, 4, \dots\}$

$$f(1) = 4(1) - 1 = 3, f(2) = 4(2) - 1 = 7, f(3) = 4(3) - 1 = 11, f(4) = 4(4) - 1 = 15, \dots$$

$$\boxed{3, 7, 11, 15, \dots}$$

6) Describe the **domain** and **range** of a function that might describe the sequence of $\{14, 11, 8, 5, \dots\}$

$$\text{DOMAIN: } \{1, 2, 3, 4, \dots\}$$

$$a_1, a_2, a_3, a_4$$

$$\text{RANGE: } \{14, 11, 8, 5, \dots\}$$

7) Determine the **common difference** of the sequence and write a function that could be used to describe the sequence: $\{14, 11, 8, 5, \dots\}$

$$\text{COMMON DIFFERENCE} = 11 - 14 = -3$$

$$a_n = a_1 + (n-1)d$$

$$f(x) = 14 + (x-1)(-3)$$

$$f(x) = 14 - 3x + 3$$

$$f(x) = -3x + 17$$

$$f(x) = -3x + 17$$

8) Write a **recurrence relation** and an **explicit definition** for the following table:

n	1	2	3	4	5	
a_n	5	7	9	11	13	

$$a_1 = 5 + 0(2) = 5$$

$$a_2 = 5 + 1(2) = 7$$

$$a_3 = 5 + 2(2) = 9$$

$$a_4 = 5 + 3(2) = 11$$

$$a_5 = 5 + 4(2) = 13$$

$$a_n = a_1 + (n-1)d$$

$$a_n = 5 + (n-1)2$$

$$f(x) = 5 + (x-1)2$$

$$f(x) = 5 + 2x - 2$$

$$f(x) = 2x + 3$$

RECURRENCE RELATION:

$$a_n = a_{n-1} + 2$$

$$\text{DOMAIN: } \{1, 2, 3, 4, \dots\}$$

$$\text{RANGE: } \{5, 7, 9, 11, \dots\}$$

EXPLICIT DEFINITION:

$$a_n = 5 + (n-1)2$$

$$f(x) = 2x + 3$$

- 9) Write a **recurrence relation** and an **explicit definition** for the following graph:

$$1, 4, 7, 10, 13, 16, \dots$$

$$a_1 = 1 + 0(3) = 1$$

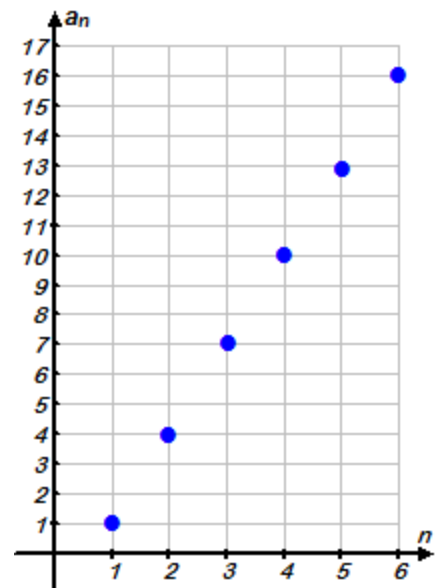
$$a_2 = 1 + 1(3) = 4$$

$$a_3 = 1 + 2(3) = 7$$

$$a_4 = 1 + 3(3) = 10$$

$$a_5 = 1 + 4(3) = 13$$

$$a_6 = 1 + 5(3) = 16$$



RECURRENCE RELATION:

$$a_n = a_{n-1} + 3$$

EXPLICIT DEFINITION:

$$a_n = 1 + (n-1) \cdot 3$$

GEOMETRIC SEQUENCES. Find the next few terms in the sequence and then find the requested term.

- 10) 3, 6, 12, 24, 48, 96, 192,

Find $a_{24} = 3 \cdot 2^{23} = 25165824$

$$a_1 = 3 \cdot 2^0 = 3$$

$$a_2 = 3 \cdot 2^1 = 6$$

$$a_3 = 3 \cdot 2^2 = 12$$

$$a_4 = 3 \cdot 2^3 = 24$$

$$a_5 = 3 \cdot 2^4 = 48$$

$$a_6 = 3 \cdot 2^5 = 96$$

$$a_7 = 3 \cdot 2^6 = 192$$

$$3 \cdot 2^{23}$$

$$25165824$$

RECURRENCE RELATION:

$$a_n = 2 \cdot a_{n-1}$$

EXPLICIT DEFINITION:

$$a_n = 3 \cdot 2^{n-1}$$

- 11) 2, -6, 18, -54, 162, -486, 1458,

Find $a_{16} = 2 \cdot (-3)^{15} = -28697814$

$$a_1 = 2 \cdot (-3)^0 = 2$$

$$a_2 = 2 \cdot (-3)^1 = -6$$

$$a_3 = 2 \cdot (-3)^2 = 18$$

$$a_4 = 2 \cdot (-3)^3 = -54$$

$$a_5 = 2 \cdot (-3)^4 = 162$$

$$a_6 = 2 \cdot (-3)^5 = -486$$

$$a_7 = 2 \cdot (-3)^6 = 1458$$

$$r = \frac{a_2}{a_1} = \frac{-6}{2} = -3$$

$$2 \cdot (-3)^{15} = -28697814$$

EXPLICIT DEF. OF
GEOMETRIC SEQUENCE

$$a_n = a_1 \cdot r^{n-1}$$

RECURRENCE RELATION:

$$a_n = -3 \cdot a_{n-1}$$

EXPLICIT DEFINITION

$$a_n = 2 \cdot (-3)^{n-1}$$

12) Create a sequence based on the function: $f(x) = 5 \cdot 2^x$

$\overbrace{f(1) = 5 \cdot 2^1 = 10}^{x=1}$,
 $\overbrace{f(2) = 5 \cdot 2^2 = 20}^{x=2}$,
 $\overbrace{f(3) = 5 \cdot 2^3 = 40}^{x=3}$,
 $\overbrace{f(4) = 5 \cdot 2^4 = 80}^{x=4}$,
 160, ...

13) Write a **recurrence relation** and an **explicit definition** for the following table:

n	1	2	3	4	5	
a_n	2	10	50	250	1250	

RECURRENCE RELATION:

$$a_n = 5 \cdot a_{n-1}$$

EXPLICIT DEFINITION:

$$a_n = 2 \cdot 5^{n-1}$$

14) Write a **recurrence relation** and an **explicit definition** for the following graph:

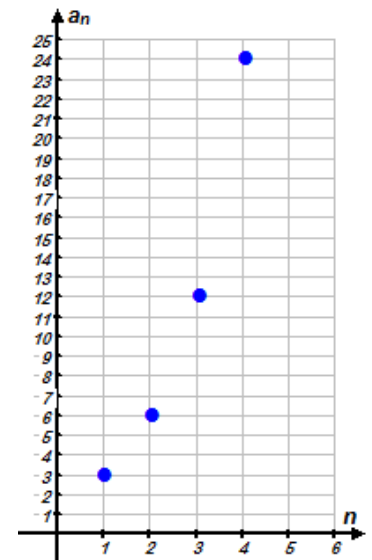
$3, 6, 12, 24$
 $a_1 = 3 \cdot 2^0 = 3$
 $a_2 = 3 \cdot 2^1 = 6$
 $a_3 = 3 \cdot 2^2 = 12$
 $a_4 = 3 \cdot 2^3 = 24$

RECURRENCE RELATION:

$$a_n = 2 \cdot a_{n-1}$$

EXPLICIT DEFINITION:

$$a_n = 3 \cdot 2^{n-1}$$



SEQUENCES

15) Given that a sequence is **arithmetic**, $a_1 = 5$, and the common difference is 4, find a_{37} .

ARITHMETIC: $a_n = a_1 + (n-1)d$

$$a_{37} = 5 + (37-1)4$$

$$a_{37} = 5 + (36)4$$

$$a_{37} = 5 + 144$$

$$a_{37} = 149$$

16) Given that a sequence is **arithmetic**, $a_{52} = 161$, and the common difference is 3, find a_1 .

ARITHMETIC: $a_n = a_1 + (n-1)d$

$$161 = a_1 + (52-1)3$$

$$161 = a_1 + (51)3$$

$$161 = a_1 + 153$$

$$\begin{array}{r} 161 \\ -153 \\ \hline 8 = a_1 \end{array}$$

$$8 = a_1$$

17) Given that a sequence is **geometric**, the first term is 1536, and the common ratio is $\frac{1}{2}$, find the 7th term in the sequence.

GEOMETRIC: $a_n = a_1 \cdot r^{n-1}$

$$a_7 = (1536) \left(\frac{1}{2}\right)^{7-1}$$

$$a_7 = (1536) \left(\frac{1}{2}\right)^6$$

$$a_7 = 24$$

18) Given that a sequence is **geometric**, $a_{10} = 98415$, and the common ratio is 3, find a_1 .

GEOMETRIC: $a_n = a_1 \cdot r^{n-1}$

$$98415 = a_1 \cdot 3^{10-1}$$

$$98415 = a_1 \cdot 3^9$$

$$98415 = 19683 a_1$$

$$\begin{array}{r} 98415 \\ 19683 \\ \hline 5 = a_1 \end{array}$$

$$5 = a_1$$

$$(1536) \left(\frac{1}{2}\right)^6 = 24$$

$$\begin{array}{r} 3^9 = 19683 \\ 98415 / 19683 = 5 \end{array}$$

COMMON RATIO = $100\% + 7\% = 1.07$

- 19) The value of an ounce of silver is about \$16 and over the last several years silver has increased in value by about 7%. How much should an ounce of silver be worth 20 years from now?

16, 17.12, 18.32, 19.60, 20.97, ...
 a_0 a_1 a_2 a_3 a_4

$r = 1.07$

$a_{20} = (17.12)(1.07)^{20-1} = \61.91

$a_n = a_1 \cdot r^{n-1}$

16*1.07 17.12
 Ans*1.07 18.3184
 19.60688
 20.97273616
 17.12*1.07^19
 61.9149514

- 20) A person was having a graduation party and noticed that only 5 people were there after the first hour but grew in size by 61% every hour. If the size of the party grew this way for 6 hours, how many people would be at the party on the 6th hour? COMMON RATIO = $100\% + 61\% = 1.61$

5, 8, 13, 21, 34, 54, ...
 a_1 a_2 a_3 a_4 a_5 a_6

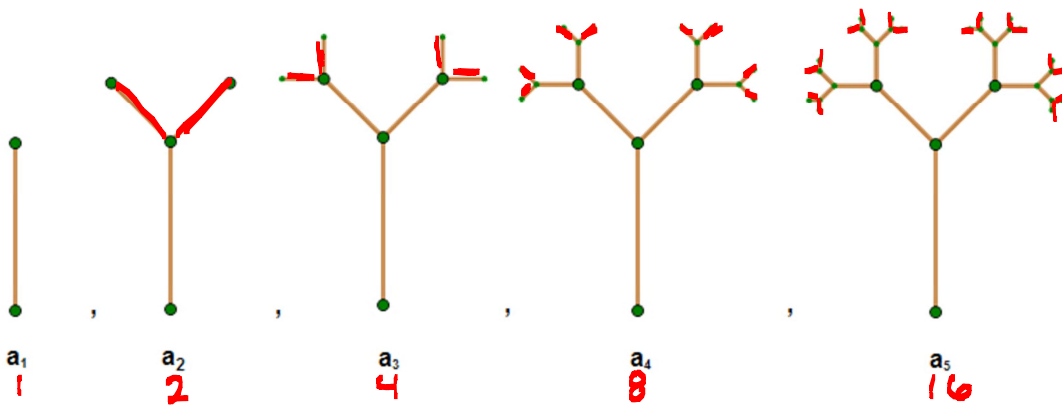
$a_n = 5 \cdot (1.61)^{n-1}$
 $= 5 \cdot (1.61)^{6-1} \approx 54$

$a_n = a_1 \cdot r^{n-1}$

5*1.61 8.05
 Ans*1.61 12.9605
 20.866405
 33.59491205
 54.0878084



- 21) Jessica is creating a drawing on her paper called a Binary Tree.



If Jessica continue drawing more and more branches, how many new branches would she need to draw on the 12th step?

1, 2, 4, 8, 16
 a_1 a_2 a_3 a_4 a_5

$r = \frac{2}{1} = 2$

$a_n = a_1 \cdot r^{n-1}$
 $a_{12} = (1)(2)^{12-1}$
 $= 1 \cdot 2^{11} = 2048$

$= 2048$